On the Nature of Angular Momentum Transport in Nonradiative Accretion Flows

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ABSTRACT

The principles underlying a proposed class of black hole accretion models are examined. The flows are generally referred to as "convection-dominated," and are characterized by inward transport of angular momentum by thermal convection and outward viscous transport, vanishing mass accretion, and vanishing local energy dissipation. In this paper we examine the viability of these ideas by explicitly calculating the leading order angular momentum transport of axisymmetric modes in magnetized, differentially rotating, stratified flows. The modes are destabilized by the generalized magnetorotational instability, including the effects of angular velocity and entropy gradients. It is explicitly shown that modes that would be stable in the absence of a destabilizing entropy gradient transport angular momentum outwards. There are no inward transporting modes at all unless the magnitude of the (imaginary) Brunt-Väisälä frequency is comparable to the epicyclic frequency, a condition requiring substantial levels of dissipation. When inward transporting modes do exist, they appear at long wavelengths, unencumbered by magnetic tension. Moreover, very general thermodynamic principles prohibit the complete recovery of irreversible dissipative energy losses, a central feature of convection-dominated models. Dissipationless flow is incompatible with the increasing inward entropy gradient needed for the existence of inward transporting modes. Indeed, under steady conditions, dissipation of the free energy of differential rotation inevitably requires outward angular momentum transport. Our results are in good agreement with global MHD simulations, which find significant levels of outward transport and energy dissipation, whether or not destabilizing entropy gradients are present.

Subject headings: accretion — accretion disks — black hole physics — convection — instabilities — (magnetohydrodynamics:) MHD — turbulence

1. Introduction

Originally developed to be powerful luminosity sources, black hole accretion models are now confronted by an embarrassing plethora of underluminous X-ray emitters, the best known of which is the Galactic center source Sgr A* (Melia & Falcke 2001). These low luminosity objects are thought to be prime candidates for a class of theoretical accretion models that has been intensively studied in recent years, which we shall refer to generically as nonradiative accretion flows. Accretion generally requires significant energy loss, and the absence of radiative losses in these flows means that determining the ultimate fate of the gas is less than straightforward.

Much of the recent interest in nonradiative flows was sparked by the work of Narayan & Yi (1994), who examined a series of one-dimensional, self-similar, steady accretion models referred to as "advection-dominated accretion flows," or ADAFs for short. In these models an α viscosity allows angular momentum transport and the resulting flows are quasi-spherical and substantially sub-Keplerian. Dissipative heating increases toward the flow center, creating an inwardly increasing entropy profile which we shall refer to as "adverse."

Nonradiative accretion flows are amenable to numerical simulation. Hydrodynamical simulations carried out with large assumed α values (> 0.3) show some similarities to ADAFs (Igumenschev & Abramowicz 1999, 2000), but smaller values of α led to flows with substantial turbulence, smaller than anticipated net inward mass accretion rates, and density distributions that are far less centrally peaked than in ADAFs (Stone, Pringle, & Begelman 1999; Igumenschev & Abramowicz 1999, 2000). Both inward and outward mass fluxes were observed at different times and different locations within the flow, and they nearly cancelled.

These findings were given the following interpretation by Narayan, Igumenschev, & Abramowicz (2000; hereafter NIA), Quataert & Gruzinov (2000; hereafter QG), and Abramowicz et al. (2002; hereafter AIQN). The adverse entropy gradient triggers an instability, and significant levels of convection result, hence the global solutions were called "convection-dominated accretion flows" (CDAFs). The next step in the argument is key: invoking the findings of other hydrodynamical simulations (Stone & Balbus 1996; Igumenschev, Abramowicz, & Narayan 2000), the angular momentum transport generated by the convective turbulence was claimed to be *inward*. This inward transport by convection was envisioned to be sufficiently great as to cancel the primary outward angular momentum transport by whatever process the α viscosity was modeling—presumably the magnetorotational instability, or MRI (Balbus & Hawley 1991).

In the CDAF scenario the vanishing of the angular momentum flux implies that the $R\phi$ component of the stress tensor responsible for accretion also vanishes (NIA, AIQN). This has the further consequence that there is essentially no mass accretion and no dissipation, despite the presence of vigorous turbulence throughout the bulk of the flow. The only region where there is any mass accretion in the model is at the very inner edge of the flow. All of the energy release associated with this small net accretion is transported outward to infinity by the surrounding convective flow which is maintained with no further dissipative losses. For this reason, CDAFs are put forth as natural candidates to explain under-luminous X-ray sources. These models have been elaborated upon, becoming influential and widely-cited. Since black hole accretion models are central to our understanding of much of X-ray astronomy, the theoretical foundations for CDAFs deserve careful scrutiny.

In this work we carry out an explicit analysis of magnetized, rotationally-supported gas in the presence of an adverse entropy gradient. We find that CDAF models have two major inconsistencies. First, locally unstable disturbances with adverse entropy gradients do not generally transport angular momentum inwards in magnetized fluids. Rather, they generally transport angular momentum outwards. Qualitatively, their behavior is indistinguishable from standard MRI modes. This latter point has been emphasized elsewhere (Hawley, Balbus, & Stone 2001; Balbus 2001), but here we demonstrate it quantitatively by explicitly calculating the leading order angular momentum transport associated with unstable WKB modes. Convective modes transport angular momentum outwards when magnetic tension is significant, and inwards only for the very longest wavelength (global scale) disturbances, where magnetic tension forces are negligible. Indeed, for a given wavenumber, the direction of angular momentum transport is less a matter of whether it is destabilized by convection or rotation, and more a matter of the nature of background medium: is it effectively magnetized or not? This is the crucial issue.

The second difficulty is more direct and fundamental, affecting magnetic and nonmagnetic models alike. By relying upon dissipated heat energy to trigger a convective instability that supposedly renders the flow dissipation-free, CDAFs run afoul of thermodynamic principles. If the source of the free energy is differential rotation, the direction of angular momentum transport *must* be outward. This is a serious inconsistency. The dissipation is quite significant, and is indeed essential if convectively unstable entropy profiles are to be sustained.

We are led to a much more standard picture of the dynamics of turbulent accretion flows, though one at odds with the tenets of CDAF theory. The turbulent stress tensor in magnetized differentially rotating gas does *not* vanish. There is vigorous local turbulent dissipation. There is mass accretion. The near cancellation of instantaneous inward and outward mass fluxes is a property of any turbulent flow with large rms fluctuations, and not a superposition of the contributions from two distinct sources of mass flux with opposite signs.

In the following sections, we present (§2) the details of the angular momentum calculation showing outward transport; (§3) an explanation of important thermodynamic inconsistencies evident in CDAF theory; (§4) a brief review of numerical simulations and a concluding summary.

2. Radial Angular Momentum Transport

2.1. Local WKB Modes

Consider a disk with radially decreasing outward entropy and pressure gradients. We use standard cylindrical coordinates (R, ϕ, Z). The square of the Brunt-Väisälä frequency (N^2) is thus negative, and tends to destabilize. In what follows, it is convenient to work with the positive quantity

$$\mathcal{N}^2 \equiv -N^2 = \frac{3}{5\rho}\frac{\partial P}{\partial R}\frac{\partial \ln P\rho^{-5/3}}{\partial R} > 0. \tag{1}$$

The disk is differentially rotating with decreasing outward angular velocity $\Omega(R)$, and epicyclic frequency

$$\kappa^2 = 4\Omega^2 + \frac{d\Omega^2}{d\ln R} = \frac{1}{R^3}\frac{dR^4\Omega^2}{dR} > 0. \tag{2}$$

A vertical magnetic field $\boldsymbol{B} = B\boldsymbol{e_Z}$ threads the disk. Its associated Alfvén velocity is $v_A^2 = B^2/4\pi\rho$, where ρ is the gas density. Axisymmetric WKB plane wave displacements of the form

$$\boldsymbol{\xi}(R, Z, t) = \boldsymbol{\xi}\exp(ikZ - i\omega t), \tag{3}$$

where k and ω are respectively the vertical wavenumber vector and the angular frequency, lead to the dispersion relation (Balbus & Hawley 1991)

$$\tilde{\omega}^4 + \tilde{\omega}^2(\mathcal{N}^2 - \kappa^2) - 4\Omega^2(kv_A)^2 = 0, \tag{4}$$

where

$$\tilde{\omega}^2 = \omega^2 - (kv_A)^2. \tag{5}$$

Let $\gamma = -i\omega$. Then, the unstable branch of the dispersion relation (4) is

$$\gamma^2 = -(kv_A)^2 + \frac{1}{2}\left[\mathcal{N}^2 - \kappa^2 + \sqrt{(\mathcal{N}^2 - \kappa^2)^2 + 16\Omega^2(kv_A)^2}\right]. \tag{6}$$

It is straightforward to show that these unstable modes must have

$$(kv_A)^2 < \mathcal{N}^2 - \frac{d\Omega^2}{d\ln R} = \mathcal{N}^2 + \left|\frac{d\Omega^2}{d\ln R}\right|, \tag{7}$$

and that the maximum growth rate is

$$\gamma_{max} = \frac{\Omega}{4}\left(\frac{\mathcal{N}^2}{\Omega^2} + \left|\frac{d\ln\Omega^2}{d\ln R}\right|\right), \tag{8}$$

which is attained for wavenumbers satisfying

$$(kv_A)^2_{max} = \Omega^2\left(1 - \frac{(\mathcal{N}^2 - \kappa^2)^2}{16\Omega^4}\right). \tag{9}$$

2.2. Stress Calculation

The angular momentum flux is directly related to the $R\phi$ component of the stress tensor

$$T_{R\phi} = \rho(\delta v_R\,\delta v_\phi - \delta v_{AR}\,\delta v_{A\phi}), \tag{10}$$

where δ denotes an Eulerian perturbation, and

$$\boldsymbol{\delta v_A} = \frac{\boldsymbol{\delta B}}{\sqrt{4\pi\rho}}. \tag{11}$$

For the local WKB modes we consider here, the angular momentum flux is $R\Omega T_{R\phi}$. Hence, the sign of the transport is simply the sign of $T_{R\phi}$.

The needed expressions can be written down immediately from the equations (2.3c–g) of Balbus & Hawley (1991). In terms of γ, they are

$$\delta v_\phi \delta v_R = (\delta v_R^2) \frac{\Omega}{D\gamma} \left(\frac{(kv_A)^2}{\gamma^2} \left| \frac{d\ln\Omega}{d\ln R} \right| - \frac{\kappa^2}{2\Omega^2} \right), \tag{12}$$

$$-\delta v_{A\phi} \delta v_{AR} = (\delta v_{AR})^2 \frac{2\Omega}{D\gamma} = (\delta v_R)^2 \left(\frac{kv_A}{\gamma} \right)^2 \frac{2\Omega}{D\gamma}, \tag{13}$$

where

$$D = 1 + \frac{(kv_A)^2}{\gamma^2}. \tag{14}$$

These equations are general beyond our simple example, holding in the presence of both vertical and radial entropy gradients. Note that there is no explicit dependence upon $\mathcal{N}$; the only dependence upon $\mathcal{N}$ anywhere is through the growth rate γ. The Maxwell stress (13) must always be positive. For a given growth rate, angular momentum transport is completely determined by rotation and magnetic tension.

Figure (1) shows the stability and angular momentum transport properties of an unmagnetized Keplerian disk in the $k^2 - \mathcal{N}^2$ plane. The vertical scale is immaterial: when $\mathcal{N} > \kappa$, convectively unstable modes are triggered at all wavenumbers. There is nothing special about the physics of convection *per se* with regard to inward angular momentum transport. In the absence of a magnetic field, any axisymmetric disturbance governed by the above equations would transport angular momentum inwards. This is why hydrodynamic simulations consistently find inward transport.

Matters change radically when a magnetic field is present. By combining equations (6), (12), and (13), we may calculate the full range of wavenumbers that transport angular momentum outwards:

$$(kv_A)^2 > \frac{1}{16} \left(4 - \left| \frac{d\ln\Omega^2}{d\ln R} \right| \right) \left(2\mathcal{N}^2 - \kappa^2 \right). \tag{15}$$

It is apparent that unless $\mathcal{N}$ is sustained at values in excess of $\kappa/\sqrt{2}$, every mode, convective or otherwise, will transport angular momentum outward.

The content of equation (15) is shown graphically in figure (2). Clearly, the dominant direction of angular momentum transport is now outwards. Note that the sense of transport is outward even for those wavenumbers that satisfy the condition

$$\left| \frac{d\Omega^2}{d\ln R} \right| < (kv_A)^2 < \mathcal{N}^2 + \left| \frac{d\Omega^2}{d\ln R} \right| \tag{16}$$

which are unstable *only* because of the presence of an adverse entropy gradient. These wavelengths occupy Region A in the figure, specifically the shaded wedge above the dotted line $(kv_A)^2 = 3\Omega^2$. These wavelengths would otherwise be stable to the "pure MRI." They

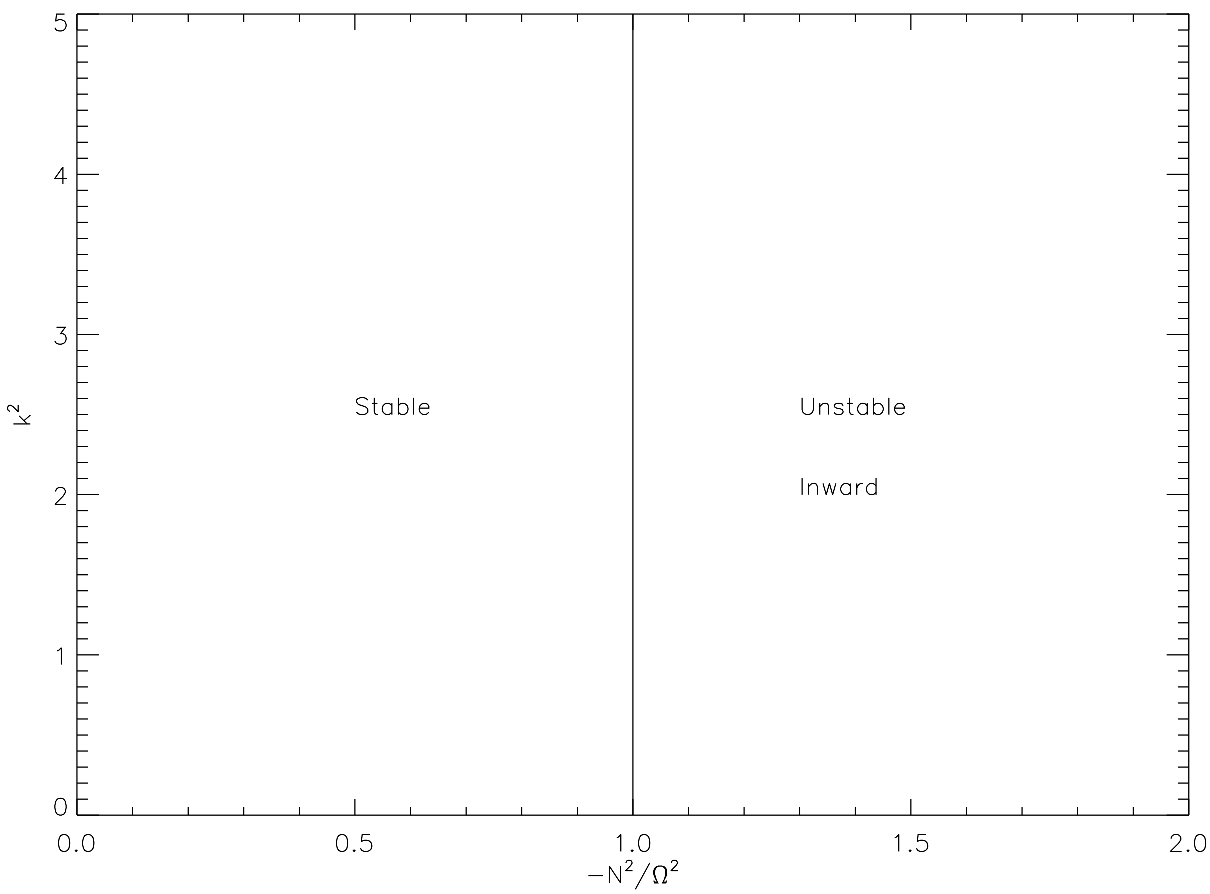

Fig. 1.— Region of instability in the $k^2 - |N^2|/\Omega^2$ plane, for an unmagnetized disk with a Keplerian rotation profile. The system is unstable for all wavenumbers when $|N^2| > \kappa^2$, and the sense of transport is inward.

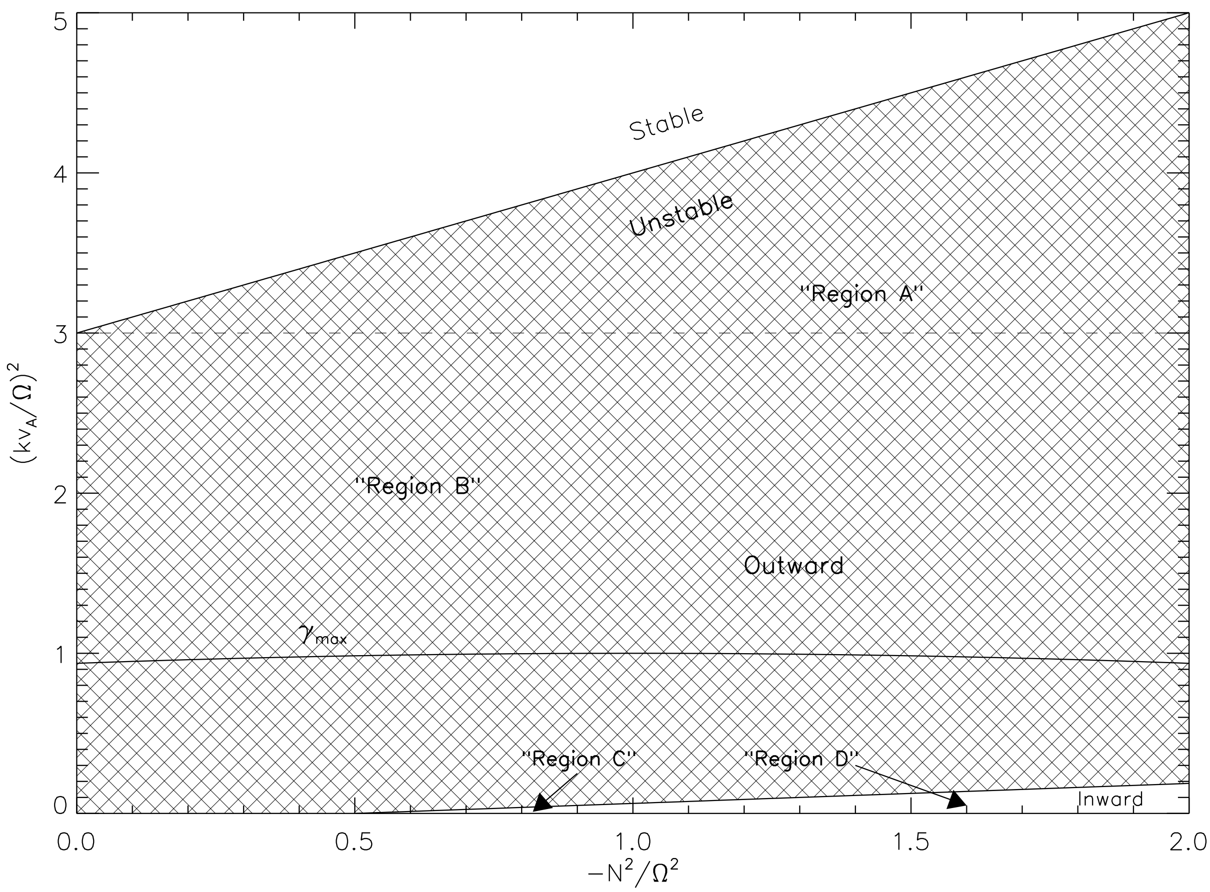

Fig. 2.— Region of instability in the $(kv_A/\Omega)^2$ – $|N^2|/\Omega^2$ plane, for a magnetized disk obeying a Keplerian rotation law. (Other rotation profiles lead to qualitatively similar diagrams.) The hatched region indicates the domain of outward transport. The solid line labeled γ_{max} shows the wavenumber of maximum growth rate as a function of $\mathcal{N}^2$. The thin wedge at the bottom corresponds to the inward transport region. See the text for the definitions of regions A through D.

are destabilized only because of the existence of adverse entropy gradients, yet all region A modes transport angular momentum outwards. Physically this is because the large magnetic tension, which regulates angular momentum transport in this regime, would oridinarily be a strongly stabilizing agent. The adverse entropy gradient destabilizes, but does not alter the outward flow of angular momentum.

Region B denotes the shaded area below region A, but above the unshaded wedge comprising regions C and D. The B region is the wavenumber domain of outward transporting modes that would be unstable in the presence of the pure MRI. Their growth rate is increased by the presence of finite $\mathcal{N}^2$. The curve showing the most rapidly growing wavenumber (equation [9]) lies in region B, as shown, but eventually leaves this region at larger values of $\mathcal{N}^2$ (see below).

The inward transporting modes are confined to the unshaded narrow wedge at the bottom of the diagram, corresponding to long wavelengths. These modes have only a small magnetic tension force, $(kv_A)^2 \ll \Omega^2$. Region C consists of modes with $\mathcal{N}^2 < \kappa^2$. Comparison with figure (1) shows that region C modes are destabilized only by the MRI (i.e., they are stable in hydro disks), but are nevertheless associated with inward transport. Finally, region D identifies the modes that would be unstable in a purely hydrodynamic system that likewise transport angular momentum inward. In this narrow domain, both magnetic tension and rotational stabilization are smaller than the adverse entropy gradient. The curve denoting the most rapidly growing wavenumber enters region D for values of $\mathcal{N}^2$ in excess of $4\Omega^2$.

The region of inward transport is very small, indeed non-existent for $\mathcal{N}^2 < 0.5\kappa^2$, and it would be surprising if these marginally-valid WKB modes effectively halted angular momentum transport in nonradiative flows. In the next section we show that this is in fact impossible in any system where the seat of free energy is differential rotation. For the present, it is useful to have an estimate (or a bound) of the size of $\mathcal{N}^2$ one expects in nonradiative flows.

The entropy equation for a monotomic gas is

$$\frac{3}{2}P\frac{d}{dt}\left(\ln P\rho^{-5/3}\right) = Q^+ \tag{17}$$

where Q^+ represents dissipative heating. This will generally not exceed the total energy budget available from differential rotation, $-T_{R\phi}d\Omega/d\ln R$, and may be much less. In a one-dimensional approximation, we therefore expect

$$\frac{3}{2}Pv_r\frac{d}{dr}\left(\ln P\rho^{-5/3}\right) \lesssim -T_{R\phi}\frac{d\Omega}{d\ln R} \tag{18}$$

Let us work near the equatorial plane and switch to cylindrical radius R. The inward drift velocity is related to the stress tensor by (Balbus & Hawley 1998):

$$v_R \simeq -\frac{T_{R\phi}}{\rho R\Omega}, \tag{19}$$

which leads to

$$\frac{1}{\rho}\frac{d}{dR}\left(\ln P\rho^{-5/3}\right) \lesssim \frac{1}{3}\frac{R}{P}\frac{d\Omega^2}{d\ln R}, \tag{20}$$

and

$$\mathcal{N}^2 = \frac{3}{5\rho}\frac{dP}{dR}\frac{d}{dR}\left(\ln P\rho^{-5/3}\right) \lesssim \frac{1}{5}\frac{d\ln P}{d\ln R}\frac{d\Omega^2}{dR}. \tag{21}$$

Generally, when differential rotation increases, the pressure gradient decreases, and vice-versa. For a Keplerian profile, $\mathcal{N}^2/\Omega^2 \lesssim 0.6 d\ln P/d\ln R$. In such a disk, pressure gradients are small, and $\mathcal{N}^2/\Omega^2$ is likely to be less than unity. If a significant fraction of free energy goes into generating a magnetic field that is carried off from the disk (i.e., not into dissipative field reconnection), then $\mathcal{N}^2$ could be appreciably smaller. The point here is that a well-defined bound on the rate of energy dissipation limits the size of $\mathcal{N}^2$.

3. Theoretical Implications

To the extent that an MHD fluid description is valid, the stability of black hole accretion flow is regulated by the generalized Høiland criteria presented in Balbus (1995) or Balbus (2001). The notion of a "convective mode" is ambiguous, as evidenced in figure (2). A perturbation in a given background flow is best characterized simply by its wavenumber. The important physical point is not to categorize and study convective modes, but to understand that magnetic tension in an accretion flow produces outward angular momentum transport.

3.1. Turbulence and Irreversibility

The dissipative properties of CDAFs are very striking, and, in light of the results of §2, they merit reexamination. In a CDAF (NIA, AIQN), outward angular momentum is driven by a viscous-like primary instability in the fluid, and the energy from the differential rotation is dissipated as heat. This heating creates an adverse entropy gradient which, in turn, generates a convective-like secondary instability. When both the primary viscous and secondary convective instabilities are active, their angular momentum transport is equal and opposite, and the total angular momentum flux drops to zero, as does the volume-specific energy dissipation rate Q^+. This means that the convective instability recovers the dissipated heat (that originally produced the convective instability), and returns it to the fluid in the form of work (inward angular momentum and outward energy fluxes).

Clearly, this is a violation of the second law of thermodynamics, whether the system is hydrodynamical or magnetohydrodynamical: the onset of convection is caused by irreversible heat dissipation, and this energy can not be fully recovered in the form of work. NIA identify

$$-T_{R\phi}\frac{d\Omega}{d\ln R} \tag{22}$$

as the *fundamental* expression for the volume specific dissipative energy loss rate. But the fundamental definition of the energy loss must be in terms of the dissipation coefficients themselves. With η_v equal to the viscous diffusivity, η_B the resistivity, and $\delta\mathcal{J}$ the

fluctuating current density, the energy dissipation rate per unit volume is

$$Q^+ \equiv \sum_i \langle \eta_v |\nabla \delta v_i|^2 + \eta_B \delta \mathcal{J}^2 \rangle, \tag{23}$$

where the sum is over vector components. These losses never vanish in a turbulent cascade, and a secondary source of turbulent fluctuations can never fully recover the energy that is dissipated in the process of triggering the creation of the same source. It amounts to reversing diffusion, a thermodynamic impossibility.

This effect is not small. In a realizable nonradiative flow, none of the entropy generated via dissipation ever leaves the system (except in an outflow), for entropy can only increase. The CDAF description ignores all of the entropy production in the flow, entropy which would be required to produce the vigorous convection driving angular momentum inward. Recall that $\mathcal{N}$ needs to exceed $\kappa/\sqrt{2}$ for there to be any unstable modes transporting angular momentum inward, and even this minimum threshold already requires substantial levels of energy dissipation to be present. The description of a CDAF as a dissipation-free flow (or even nearly so) lacks a fundamental self-consistency.

The sources and sinks for turbulent fluctuations may be read off from Balbus & Hawley (1998), eq. (89):

$$-T_{R\phi}\frac{d\Omega}{d\ln R} + P\boldsymbol{\nabla\cdot\delta v} - Q^+. \tag{24}$$

The expression (22) happens to be equal to dissipative energy losses Q^+ under strictly defined conditions that may be inferred from the above: steady, local turbulence in which the work done by pressure is negligible. This is, in fact, a good description of the behavior of turbulence in magnetized differentially rotating flows. An immediate consequence is that the time-averaged value of $T_{R\phi}$ must be > 0, i.e.,

$$T_{R\phi} = -\left(\frac{d\Omega}{d\ln R}\right)^{-1} Q^+ > 0. \tag{25}$$

The unavoidable presence of dissipation compels a net outward angular momentum flux in any time-steady magnetized system in which the free energy source is differential rotation.

4. Comparison With Simulations

The point of contact of CDAFs with numerical simulations is the power law scaling of the density, $\rho(r)$, where r is spherical radius. The argument runs that since the CDAF energy flux is conserved (no dissipation), $\rho v^3 r^2$ is a constant. With $v \sim r^{-1/2}$, this gives $\rho \sim r^{-1/2}$ as well. By way of contrast, constancy of the mass flux $\sim \rho v r^2$ would give a much steeper $\rho \sim r^{-3/2}$ power law, which is associated with an ADAF solution. The power law scaling $\rho \sim r^{-1/2}$ has been interpreted as evidence in support of convection-dominated flows.

Since gas accreting into a black hole is almost certainly magnetized at some level, there is very little to be gained by hydrodynamical simulations: the stability and transport

properties of magnetized and unmagnetized gases are simply too different from one another. In general, MHD simulations have not been supportive of CDAFs (Stone & Pringle 2000; Hawley, Balbus, & Stone 2001; Hawley & Balbus 2002). The one exception cited is that of Machida, Matusmoto and Mineshige (2001). This simulation follows the evolution of a magnetized torus in a Newtonian potential. "Convective motions" are observed in the subsequent accretion flow. However, these authors draw no distinction between what they refer to as convective motions and turbulence in general. Turbulence is, of course, inevitable in an MHD accretion flow; it is hardly the defining signature of a CDAF. The only quantitative basis for the claimed agreement with CDAFs was the observation that at one point in time the the density exhibited a $r^{-1/2}$ power law behavior. However, the flow was a highly time-dependent, the r dependency evolved, and a different power law emerged at the end of the simulation.

Even if $\rho \sim r^{-1/2}$ scaling were unambiguously extracted from a simulation, there is no compelling association with thermal convection. Under steady conditions, all nonradiative flows, regardless of their level of dissipation, have a conserved mass flux and a conserved total energy flux. When the flow is turbulent, to address the behavior of these fluxes one must speak in terms of correlations of the flow quantities. In the presence of fluctuations, it is common to have weak correlations in the mass flux and much stronger correlations in the energy flux; indeed, ordinary waves display this sort of behavior. The scaling $\rho \sim r^{-1/2}$ could emerge, for example, if the correlation between pressure and velocities in the energy flux were strong, and the magnitude of all velocity fluctuations scales by the local virial velocity. Neither the presence or absence of thermal convection, nor the rate of energy dissipation is relevant.

In any case, any similarity between theory and simulation in a radial power law index is not an answer to fundamental dynamical and thermodynamical inconsistencies. Verification should run in the opposite direction: in the course of producing MHD turbulence, any numerical simulation should show on average a positive value for $T_{R\phi}$, and significant levels of energy dissipation.

The notion of a convectively unstable mode is ill defined in magnetized differentially rotating systems. When the magnitude of the Brunt-Väisälä frequency $\mathcal{N}$ is less than $\kappa/\sqrt{2}$, all unstable modes transport angular momentum outward. If $\mathcal{N}$ is maintained above this, equation (15) and figure (2) show that a small range of wavelengths on the largest scales transport angular momentum inwards. (It is of course not at all surprising that sufficiently large values of $\mathcal{N}$ can be chosen to render the dynamics of differential rotation less important than convection.) For $\mathcal{N} < \kappa$, the inward transporting modes are destabilized by magnetic tension, for larger $\mathcal{N}$ values the destabilization is by convection. Small wavenumber convective modes (Region A in figure [2]) always transport angular momentum outwards, and overall transport remains overwhelmingly outwards for values of $\mathcal{N} \sim \Omega$.

This is a very serious difficulty for CDAF models, which rely on a significant level of inward angular momentum transport engendered by convectively-driven modes. Moreover, and most importantly, the claims of vanishing stress and vanishing energy dissipation are inconsistent with fundamental thermodynamic principles. They are also inconsistent with the need to maintain $\mathcal{N} > \kappa/\sqrt{2}$, a requirement for the existence of any inward transporting unstable modes. The central feature of CDAF theory, a vanishing of the angular momentum flux, is not possible if the seat of free energy is differential rotation. Characteristic power law scalings extracted from numerical simulations are not evidence for low dissipation flow.

Black hole accretion features far more conventional and familiar fluid dynamics. Underluminous sources are likely to be a consequence of low densities and lower than anticipated temperatures, as opposed to dissipation-free turbulence. MHD turbulence leads to outward angular momentum transport and to a positive $R\phi$ stress component, irreversible dissipative heating, mass accretion, and significant mass outflow as well. These are all manifest in numerical MHD simulations. The development of the spectral and energetic properties will test these model flows more stringently.

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